J. Fluid Mech. (1973), *vol.* 61, *part* 1, *pp.* 159–172
Printed in Great Britain

The steady two-dimensional reflexion of an oblique partly dispersed shock wave from a plane wall

By H. BUGGISCH
Fachbereich Mechanik, Technische Hochschule, Darmstadt, Germany

(Received 16 January 1973)

The steady two-dimensional problem of reflexion of an oblique partly dispersed plane shock wave from a plane wall is studied analytically. Viscosity, diffusion and heat conduction are neglected. The thermodynamic state of the gas is assumed to be determined by the instantaneous values of the specific entropy s, pressure p and a finite number of internal state variables. Results for the flow field behind the reflected shock are obtained by a perturbation method which is based on the assumption that the influence of relaxation is relatively weak.

1. Introduction

Recent studies of one-dimensional unsteady shock reflexion in a relaxing gas have shown that the wall pressure history is very sensitive to the relaxation processes that occur in the gas. Therefore, shock reflexion is a very useful research tool for the experimental determination of reaction rates (see, for example, Baganoff 1965; Johannesen, Bird & Zienkiewicz 1967; Smith 1968; Buggisch 1970; Becker 1972). Unfortunately, this method has certain disadvantages: for instance, the time during which useful data may be obtained is rather short, namely of the order of a few μs. Hence, it is necessary to use very fast reacting instruments. This disadvantage could be avoided if, instead of the one-dimensional flow, one could use a suitable steady flow which is equally sensitive to relaxation processes. Such a flow can indeed be found, namely the two-dimensional steady flow of the gas through an oblique shock which is reflected from a plane wall. Of course, the very short time available in unsteady flow is now replaced by a very short distance. This being so, the steady flow will be preferable for an experimentalist only if high space resolution is obtainable. Furthermore, serious difficulties will arise from interactions between the shock wave and the boundary layer if the flow field is produced by the reflexion of a shock from a wall. Fortunately, the same flow field without a boundary layer is established by two oblique shocks crossing each other symmetrically. Even though there may be some doubt whether the steady reflexion is really useful for an experimentalist, we think that the problem is interesting in its own right and therefore should be investigated.

As we have mentioned, the two-dimensional steady reflexion is sensitive to relaxation processes in a similar way to the one-dimensional unsteady reflexion.

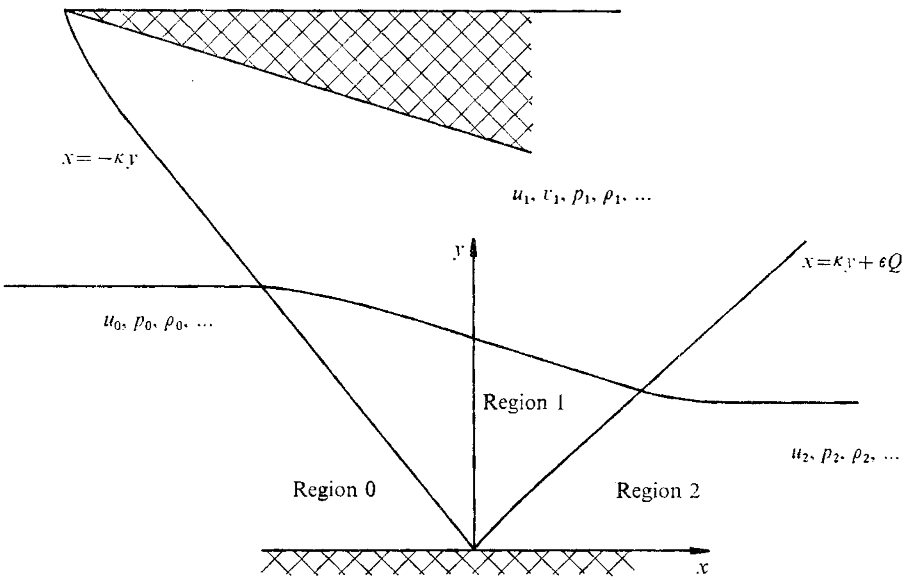

FIGURE 1. Sketch of the flow field under consideration.

This is plausible: in both cases the gas particles pass two shocks. For particles which are far away from the wall, in both cases there is enough time for them to relax to the equilibrium state before they meet the reflected shock. In contrast, in both cases particles near the wall pass the reflected shock before the relaxation processes induced by the incident shock are finished.

In what follows, we shall present a theory of steady two-dimensional reflexion of a partly dispersed oblique shock wave in a relaxing gas where viscosity and heat conduction are neglected. To our knowledge, this problem has not yet been considered.

2. Description of the flow field and basic equations

Figure 1 shows the flow field qualitatively. A steady oblique shock of given inclination, which, for example, can be produced by a wedge, meets at $x = 0$ the solid plane wall $y = 0$. The gas flows from left to right. The thermodynamic state and the velocity u_0 to the left of the incident shock are given, the gas being in thermodynamic equilibrium there. At the wall, the component of velocity in the y direction must vanish. This condition can be satisfied in many cases if a reflected shock is introduced. Near the wall, the frozen reflected shock crosses the relaxation zone of the incident shock. Far away it becomes straight, the gas to the left of it in region 1 being in equilibrium. Of course, at the wedge the incident shock is also curved, but to make the theory of reflexion as simple as possible, we assume the wedge to be far away at $y = \infty$. In this case, the incident shock is given by the straight line $x = -\kappa y$ and the state of the gas in region 1 depends only on the distance from the incident shock.

The flow which we study is steady. Effects of viscosity, diffusion, heat conduction and radiation are neglected. It is assumed that body forces are unimportant.

Under these circumstances the balance equations for mass and momentum are

$$D\rho+\rho(u_x+v_y) = 0, \tag{1}$$

$$Du+\rho^{-1}p_x = 0, \tag{2}$$

$$Dv+\rho^{-1}p_y = 0 \tag{3}$$

and the integrated form of the balance of energy is

$$h=\tfrac{1}{2}(u^2+v^2) = h_0+\tfrac{1}{2}u_0^2. \tag{4}$$

The symbols used here have the following meaning: ρ = mass density, p = pressure, h = specific enthalpy and (u,v) = velocity components in the (x,y) directions. The operator D represents the material derivative.

We are concentrating on effects of thermodynamic non-equilibrium processes like vibrational relaxation or chemical reactions. Therefore, we introduce a finite number of internal state variables $\xi^1, \xi^2, \ldots, \xi^n$. These may, for example, have the physical meaning of vibrational temperatures in the case of vibrational relaxation. The internal state variables enter the theory through the equation of state

$$h = \hat{h}(p, s, \xi^1, \xi^2, \ldots, \xi^n). \tag{5}$$

To solve our problem we must know how the ξ^i change with time for given behaviour of the pressure p and the entropy s. In many cases this dependence can be given in form of a rate equation:

$$D\xi^i = L^i(p, s, \xi^1, \xi^2, \ldots, \xi^n). \tag{6}$$

In what follows, we consider only internal processes for which (6) holds.

Finally, thermodynamic considerations show that the equations

$$\hat{h}_p = \rho^{-1}, \quad \hat{h}_s = T \tag{7}$$

give the dependence of the density ρ and the temperature T on p, s and the ξ_i.

Equations (1), (2), (3) and (6) are correct only for continuous flow fields, whereas (4), (5) and (7) hold even if there are discontinuities in the field. Because we have shocks in the flow field, we must complete (1)–(7) by jump conditions for the density, for the two components of momentum and for the internal variables ξ^i. For density and momentum we take the usual jump conditions; for the ξ^i we demand simply that the ξ^i be continuous across a shock.

3. Basic assumptions for the perturbation method

The problem does not admit exact analytical solutions. Therefore, if we do not want to use a computer, we must employ some perturbation method. The method which we shall employ was used first, we think, by Spence in 1961 in his calculation of the development of the steady shock formed in a tube filled with relaxing gas into which a piston is pushed. The author himself (Buggisch 1969, 1970; see also Becker 1972) has used the same method for the problem of one-dimensional unsteady shock reflexion. The basic assumption is that the influence of relaxation on the flow field is relatively weak. To be more specific, we assume that the enthalpy and its first and second derivatives with respect to p, s and ξ^i ($i = 1, 2, ..., n$) never differ much from the equilibrium enthalpy and its derivatives. In other words, we assume that we can split $\hat{h}$ into two parts $\tilde{h}$ and $\epsilon\hat{H}$, where $\tilde{h}$ is the equilibrium enthalpy and where $\epsilon > 0$ is a small number which measures the influence of non-equilibrium processes on the flow field:

$$\hat{h} = \tilde{h}(p, s) + \epsilon\hat{H}(p, s, \xi^1, \xi^2, ..., \xi^n). \tag{8}$$

Further, we assume that $\hat{H}$ and its first and second derivatives are, at most, of the order of $\tilde{h}$ and its derivatives in the entire flow field. Of course, in a thermodynamic equilibrium state $\hat{H}$ must vanish.

The question arises as to whether there is any physical justification for the assumption that ϵ is small. We now give some reasons why there are indeed real physical situations in which ϵ is very small. First, let us consider a case where the gas is a mixture of two components, one of which is inert while the other has a finite number of relaxing internal degrees of freedom. If we choose the concentration of the relaxing component small enough, we can make ϵ as small as we wish. Second, let us take a gas with vibrational relaxation and let us assume that the temperature in the flow field remains small in comparison with the characteristic temperature for the excitation of vibrations. In this case the influence of relaxation remains very small also, i.e. ϵ is a very small number. Finally, we remark that the assumption of small ϵ is equivalent to the assumption that the difference between the frozen and equilibrium speed of sound is small. This assumption has been made in several previous papers, see, for instance, Spence (1961) and Ockendon & Spence (1969), and there is no doubt that useful and interesting results have been obtained in these papers.

To make things clearer, let us consider as an example a model gas, namely the perfect gas with one internal energy mode and with constant specific heats. The internal state variable is, in equilibrium, equal to the (translational) temperature of the gas. The canonical equation of state for this gas is (see, for instance, Becker & Böhme 1969)

$$\hat{h} = (\tilde{c}_p - R\epsilon)\, T_+ \left(\frac{p}{p_+}\right)^{R/(\tilde{c}_p - R\epsilon)} \left(\frac{\xi}{T_+}\right)^{-R\epsilon/(\tilde{c}_p - R\epsilon)} \exp\frac{s - s_+}{\tilde{c}_p - R\epsilon} + R\epsilon\xi, \tag{9}$$

where R, $\tilde{c}_p$, ϵ, T_+, p_+ and s_+ are constants, R being the gas constant, $\tilde{c}_p$ the specific heat at constant pressure of the gas at equilibrium, $R\epsilon$ the specific heat at constant pressure of the internal energy mode and T_+, p_+ and s_+ the reference temperature, pressure and entropy respectively. Now, $\hat{h}$ can be expanded in a Taylor series with respect to ϵ. If the specific heat of the internal energy mode is small, i.e. if $\epsilon \ll 1$, we can approximate this Taylor series by its first two terms:

$$\hat{h} = \underbrace{\tilde{c}_p\, T_+ \left(\frac{p}{p_+}\right)^{R/\tilde{c}_p} \exp\frac{s - s_+}{\tilde{c}_p}}_{\tilde{h}(p,s)} + \epsilon \underbrace{\left\{R(\xi - \tilde{h}_s) - \frac{\tilde{h}R}{\tilde{c}_p} \ln\frac{\xi}{\tilde{h}_s}\right\}}_{\hat{H}(p,s,\xi)}. \tag{10}$$

Equation (10) is of the same form as (8); thus we can apply the perturbation method, which we shall explain in §4, to our model gas for $\epsilon \ll 1$.

4. Development of the perturbation method and results for the general case

The perturbation method gives the solution in the form of a power series in ϵ. In the zero-order approximation the relaxation has no influence on the flow field. Thus, the velocities, the thermodynamic variables and the inclination of the reflected shock can be calculated from the shock relations and the thermodynamic properties of the gas at equilibrium. Therefore, we can consider these zero-order quantities to be known. (We remark here that we assume the situation to be such that regular reflexion, no Mach reflexion, occurs.) We denote the zero-order solution for the pressure by p_k. The subscript k refers to the region of the flow field, $k = 1$ indicating that part of the field which lies between the two shocks, and $k = 2$ the region to the right of the reflected shock (cf. figure 1). Thus p_1 denotes that constant pressure which we find for the gas at equilibrium without relaxation between the two shocks. All other quantities with subscript 1 or 2 are defined in the same way.

The set of relaxation equations in the zero-order approximation is

$$D_k \xi^i = L^i(p_k, s_k, \xi^1, \xi^2, \ldots, \xi^n), \quad i = 1, 2, \ldots, n, \quad k = 1, 2, \tag{11}$$

where the operator D_k is defined by

$$D_k = u_k \, \partial/\partial x + v_k \, \partial/\partial y.$$

We assume that we can solve this set of equations. Let the solution of (11) be

$$\xi^i = \begin{Bmatrix} \xi^i_{\mathrm{I}}(x, y) & \text{in} & \text{region 1,} \\ \xi^i_{\mathrm{II}}(x, y) & \text{in} & \text{region 2.} \end{Bmatrix} \tag{12}$$

In the case of the relaxation equation

$$D\xi = -(\xi - T)/\tau(p, s) \tag{13}$$

the solution (12) can be given explicitly:

$$\xi_{\mathrm{I}} = T_1 + (T_0 - T_1) \exp\left[-(x + \kappa y)/\Lambda_1\right],$$

$$\xi_{\mathrm{II}} = T_2 + \left\{(T_0 - T_1) \exp\left[-\frac{\kappa + \kappa'}{\Lambda_1} y\right] + T_1 - T_2\right\} \exp\left[-\frac{x - \kappa' y}{\Lambda_2}\right],$$

with $\Lambda_1 = (u_1 + \kappa v_1)\tau_1$ and $\Lambda_2 = u_2 \tau_2$. The distances Λ_1 and Λ_2 have quite a simple geometrical meaning: they are the width of the relaxation region in x direction behind the incident or reflected shock respectively.

In the last step, we shall now calculate the flow field in the first-order approximation. In this approximation the solution must be of the form

$$u = u_k + \epsilon \Delta u, \quad v = v_k + \epsilon \Delta v, \quad p = p_k + \epsilon \Delta p,$$

and so on. From this, using the balance equations, the equation of state and the thermodynamic relation $\hat{h}_p = 1/\rho$, we obtain in the first-order approximation a set of five linear equations for the five functions Δu, Δv, $\Delta \rho$, Δp and Δs.

Eliminating Δu, Δv, $\Delta\rho$ and Δs, we can reduce this system to a differential equation of second order for the pressure perturbation:

$$\left[1-\left(\frac{u_k}{a_k}\right)^2\right]\Delta p_{xx}+\left[1-\left(\frac{v_k}{a_k}\right)^2\right]\Delta p_{yy}-2\frac{u_k v_k}{a_k^2}\Delta p_{xy} = \rho_k D_k^2 r(p_k, s_k; x, y), \quad (14)$$

where
$$a_k^{-2} = \rho_k^2 \tilde{h}_{pp}(p_k, s_k) \quad (15)$$

and
$$r = \rho_k\left\{\frac{\tilde{h}_{ps}(p_k, s_k)}{T_k}\hat{H}(p_k, s_k, \xi^1(x,y), \dots, \xi^n(x,y)) - \hat{H}_p(p_k, s_k, \xi^1(x,y), \dots, \xi^n(x,y))\right\}. \quad (16)$$

a_k, as defined in (15), is the velocity of sound in the gas at equilibrium in region k. The function r (defined in (16)) depends on x and y. It can be calculated from the zero-order solution for the ξ^i. The term $D_k^2 r$ on the right-hand side of (14) represents a source term, giving the production of waves through relaxation. It vanishes in equilibrium. For our model gas, r is of the simple form

$$r = \frac{\gamma-1}{\gamma}\frac{\xi-T_k}{T_k}, \quad \gamma = \frac{\tilde{c}_p}{\tilde{c}_p-R}.$$

We consider only cases in which the flow is supersonic in region 2. In this case, (14) is hyperbolic in regions 1 and 2. The solution of (14) in region 1 can be computed easily. Because the flow is supersonic, the field can not be influenced by the presence of the reflected shock. All quantities in region 1 therefore depend only on the distance from the incident shock. This being so, the flow field can be computed even in the general case without the approximations made here. In our approximation the result is

$$\frac{1}{\rho_0 u_0^2}\Delta p \equiv \frac{1}{\rho_0 u_0^2}\Delta p_{\mathrm{I}}(x,y) = \frac{(\rho_0/\rho_1)\, r(p_1, s_1; x, y)}{1+\kappa^2-(\rho_0/\rho_1)^2(u_0/a_1)^2}.$$

In order to obtain this result, one has to make use of the relation

$$\rho_1(u_1+\kappa v_1) = \rho_0 u_0.$$

This relation simply states that the mass flux across the incident shock must be preserved.

The problem of determining the field in region 2 is much harder. Since the state of the gas to the left of the reflected shock is not constant near the wall, the reflected shock is curved and the state in region 2 does not depend only on the distance from the shock. First, we can find a general solution of (14) which satisfies the boundary condition (symmetry condition) $\partial\Delta p/\partial y = 0$ at the solid wall, i.e. at $y = 0$. This solution is

$$p_2^{-1}\Delta p = \partial\phi/\partial x + f(x+by) + f(x-by), \quad b^2 = (u_2/a_2)^2 - 1. \quad (17)$$

$\partial\phi/\partial x$ is a particular integral of the inhomogeneous differential equation for $\Delta p/p_2$ and satisfies the boundary condition $\partial^2\phi/\partial x\,\partial y = 0$ at the wall. ϕ is given by the following equations:

$$\phi = \int_\infty^x \left\{P(\bar{x}, y) - \frac{1}{b}\int_\infty^{x+by}\frac{\partial P}{\partial y}(\hat{x}, 0)\,d\hat{x}\right\}d\bar{x}, \quad (18)$$

$$P = -\frac{\rho_2 u_2^2}{4b^2 p_2}\int_\infty^{x-by}\left\{\int_\infty^{x+by} r_{xx}\,d(x+by)\right\}d(x-by). \quad (19)$$

In (19) the integrand r_{xx} is the second derivative of $r(x, y)$ with respect to x for fixed y; this integrand r_{xx} is to be considered as a function of $x+by$ and $x-by$ and to be integrated over $x+by$ and $x-by$. For $y \to \infty$, $\partial\phi/\partial x$ tends to the pressure variation in the relaxation zone of the reflected shock far away from the wall. Thus, f has to vanish for $y \to \infty$.

As an example let us consider our model gas, and let us further assume that the relaxation equation is of the form $D\xi = -(\xi - T)/\tau(p, s)$. Then we obtain for P the simple result

$$P = \left[A_1 \exp\left(-\frac{\kappa+\kappa'}{\Lambda_1} y\right) + A_2\right] \exp\left(-\frac{x-\kappa' y}{\Lambda_2}\right),$$

with

$$A_1 = (\gamma-1)\frac{(T_1/T_2)(T_0/T_1 - 1)(b^2+1)}{\{\kappa' - (\Lambda_2/\Lambda_1)(\kappa+\kappa')\}^2 - b^2},$$

$$A_2 = -(\gamma-1)\frac{(1-T_1/T_2)\Lambda_2^2}{(a_2\tau_2)^2(\kappa'^2 - b^2)}.$$

From this, using (18), we can calculate the function ϕ for the model gas without difficulty. After this short remark on the model gas, let us return to the general case again.

Using the balance of the y component of momentum and the condition that Δv must vanish far downstream, we obtain the following result for the vertical component of the velocity:

$$\frac{1}{u_2}\Delta v = -\frac{bp_2}{\rho_2 u_2^2}\left(\frac{1}{b}\frac{\partial\phi}{\partial y} + f(x+by) - f(x-by)\right). \tag{20}$$

Considering (19) and (20), we see that Δp and Δv are known if the function f is determined. We shall now calculate f by making use of the conditions which are to be satisfied at the reflected frozen shock. The form of the reflected shock will not differ much from the straight line $x = \kappa' y$, which represents the reflected shock in the zero-order approximation. In our first-order approximation we may assume it to be given by $x = \kappa' y + \epsilon Q(y)$, where Q is still unknown. At the reflected shock we must satisfy jump conditions for the components of momentum and for the fluxes of mass and energy. Using further the equation of state and the relation $\partial h/\partial p = 1/\rho$, we obtain in the first-order approximation five linear equations for the six functions Δu, Δv, Δp, $\Delta\rho$, Δs and Q to be satisfied at the reflected shock of the zero-order solution. Now, Δp and Δv are known if only f is known. Thus we have to determine five unknown functions. If Δs and Q are eliminated, which may easily be done, the following set of equations remains:

$$\mathbf{B}^{-1}\begin{bmatrix} f([\kappa'+b]y) \\ \Delta u/u_2 \\ \Delta\rho/\rho_2 \end{bmatrix} + \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} f([\kappa'-b]y) = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega^3 \end{bmatrix}. \tag{21}$$

The β_j, ω_j and the matrix $\mathbf{B}^{-1}$ are given by:

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} \rho_2/\rho_1 + 1 \\ p_2 \dfrac{\partial}{\partial p}\left(\dfrac{\rho_2}{\rho(\rho, s)} - \dfrac{T(p_1 s)}{T_2}\right) \\ (bp_2/\rho_2 u_2^2)\,\kappa'(1+\rho_2/\rho_1) \end{bmatrix}_{p=p_2,\, s=s_2},$$

with $T(p,s) = \partial\tilde{h}(p,s)/\partial s$ and $\rho^{-1}(p,s) = \partial\tilde{h}/\partial p$,

$$\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix} = \begin{bmatrix}\dfrac{\rho_0 u_0^2}{p_2}\left(1-\dfrac{\rho_2}{\rho_1}\right)+\dfrac{\rho_2}{\rho_0}\left(1-\dfrac{p_1}{p_2}\right)(1+\kappa^2)\\ 0\\ -\dfrac{u_0}{u_2}(\kappa'^2+2\kappa\kappa'-1)-\dfrac{\rho_2}{\rho_0}(1+\kappa^2)\end{bmatrix}\frac{\Delta p_{\mathrm{I}}(\kappa' y, y)}{\rho_0 u_0^2}$$

$$+\begin{bmatrix}0\\1\\0\end{bmatrix} r(p_2, s_2;\kappa' y, y) - \begin{bmatrix}\beta_1\\ \beta_2\\ 0\end{bmatrix}\frac{\partial\phi}{\partial x}(\kappa' y, y) + \begin{bmatrix}0\\0\\ \beta_3/b\end{bmatrix}\frac{\partial\phi}{\partial y}(\kappa' y, y),$$

$$\mathbf{B}^{-1} = \begin{bmatrix}\beta_1 & 2\rho_2 u_2^2/p_2 & -(1-p_1/p_2)\\ \beta_2 & -\dfrac{\rho_2 u_2^2}{T_2}\dfrac{\partial T(p,s)}{\partial p} & 1\\ -\beta_3 & \kappa'^2-\rho_2/\rho_1 & -\rho_2/\rho_1\end{bmatrix}_{p=p_2,\ s=s_2}.$$

Obviously, the ω_j are functions of y, whereas the β_j and the matrix $\mathbf{B}^{-1}$ are constant. The first of equations (21) is a linear relation between $f([\kappa'+b]y)$ and $f([\kappa'-b]y)$:

$$f([\kappa'+b]y)+Bf([\kappa'-b]y) = \omega([\kappa'+b]y), \tag{22}$$

with

$$B = B_{11}\beta_1 + B_{12}\beta_2 + B_{13}\beta_3,$$

$$\omega = B_{11}\omega_1 + B_{12}\omega_2 + B_{13}\omega_3.$$

In the solution for Δp, f describes waves reflected to and fro between the wall and the frozen shock. The constant B is nothing but the reflexion coefficient, as defined both by Lighthill (1949) and Chu (1952) for the reflexion of small amplitude waves from a shock. $|B|$ is always smaller than one. The function ω is a measure of the deviation of the jump conditions from the asymptotic jump conditions. It vanishes for $y\to\infty$. The general solution for f can easily be given in the form of an infinite series which converges for $|B| < 1$:

$$f(\eta) = \omega(\eta) - B\omega\left(\frac{\kappa'-b}{\kappa'+b}\eta\right) + B^2\omega\left(\left[\frac{\kappa'-b}{\kappa'+b}\right]^2\eta\right) \mp \ldots.$$

Here, in principle, the pressure field for the general case (where the only essential restriction is that the equation of state must be of the form of (8) with $\epsilon \ll 1$) has been calculated.

5. Results for the model gas and discussion

In this section we shall discuss results for the model gas, i.e. for the perfect gas with one internal degree of freedom and with constant specific heats which furthermore obeys the relaxation equation $D\xi = -(\xi - T)/\tau(p,s)$. One may question whether this is useful since in real gases with vibrational relaxation the vibrational specific heat is a function of temperature while the equation of state of our model gas (9) leads to constant specific heats. We give here some reasons

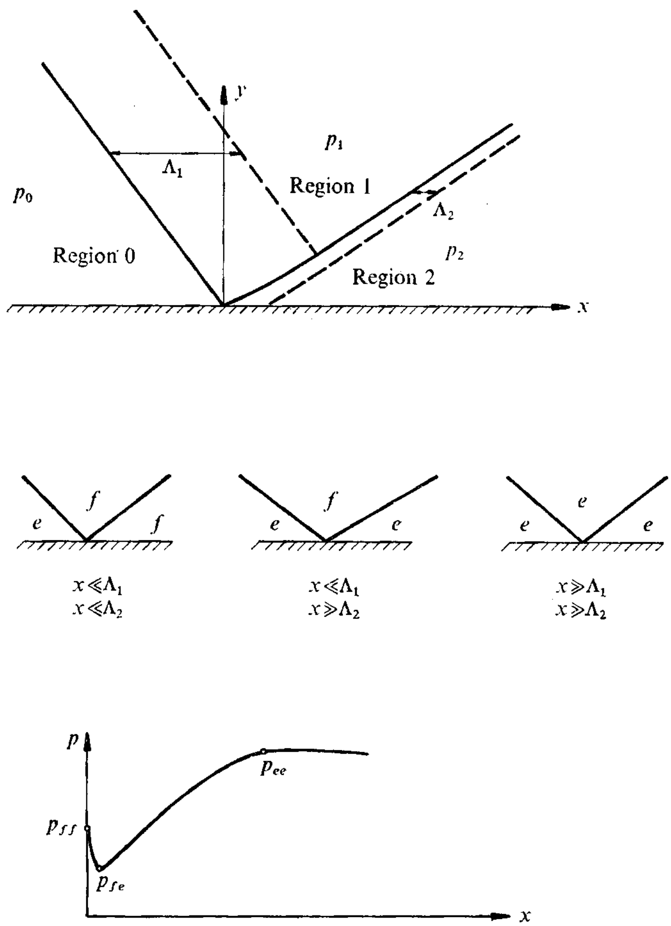

FIGURE 2. Explanation of the behaviour of the wall pressure for $\tau \ll \tau_{12}$. The subscript e stands for 'equilibrium' and f for 'frozen'.

which justify the choice of this particular model gas. First, the main aim of this paper was to develop a formalism which allows the analytical treatment of shock reflexion for the general case, in which the only essential assumption is that the equation of state is given by (8) with $\epsilon \ll 1$. The model gas is only used to illustrate some of the steps which, in principle, can be done in the general case as well but for which one needs a definite equation of state in order to get explicit results. To this end it seems best to use a model gas which leads to simple results. The perfect gas with constant specific heats seems to be very well suited for this purpose. Second, there are physical situations in which the specific heat of the internal degree of freedom is in fact nearly constant. This is especially the case for a gas with vibrational relaxation if the temperature is sufficiently high. Let us assume that the incident shock in such a gas is very strong. Then the gas behind the incident shock in region 1 is already very hot; in region 2 it is even hotter. This being so, the specific heat of the internal degree of freedom is nearly the same in both relaxation zones, while the relaxation times, depending strongly on

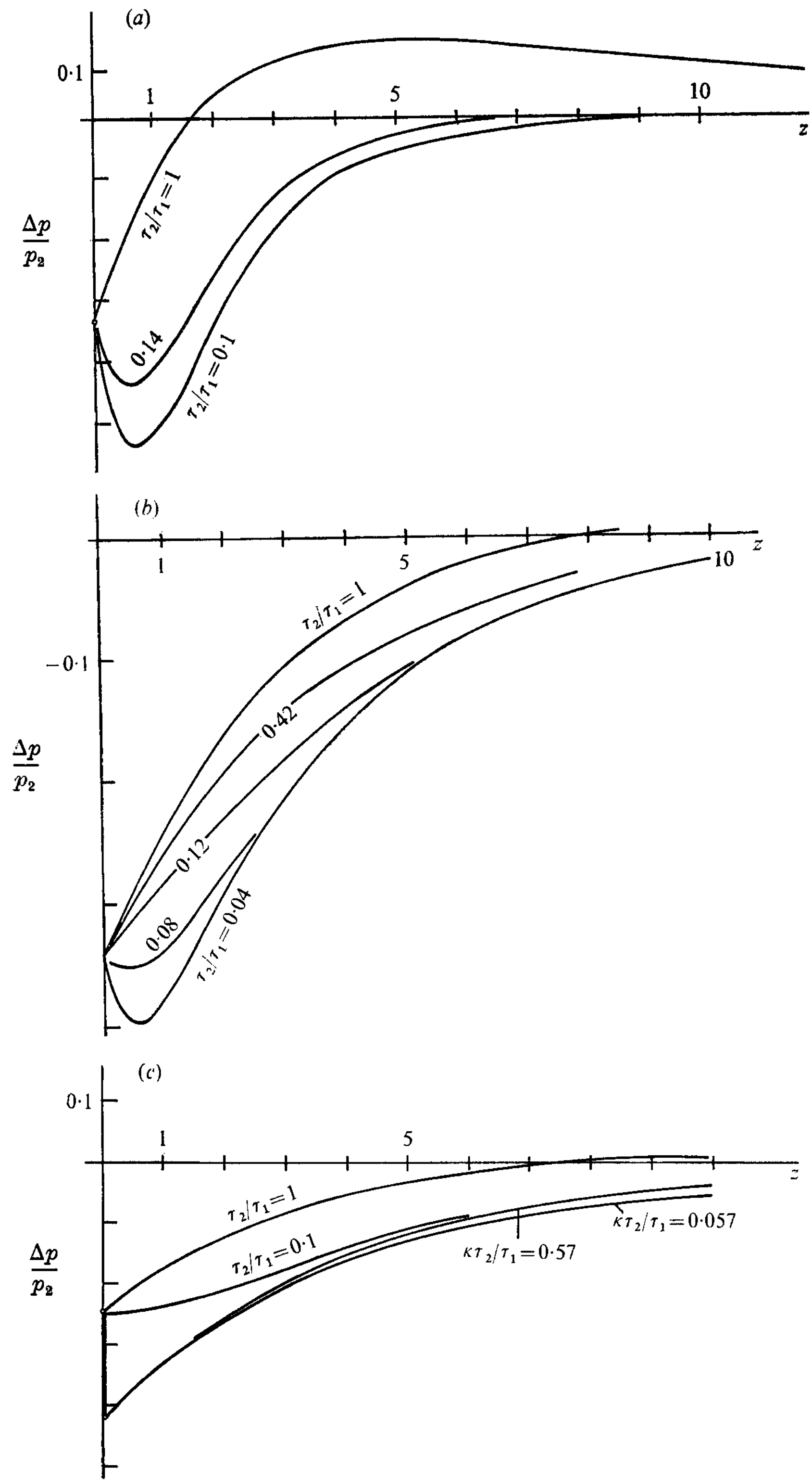

FIGURE 3. Wall pressure perturbation as a function of z for various values of the ratio of relaxation times τ_2/τ_1. (a) $\kappa = 1\cdot3$. (b) $\kappa = 2$. (c) Asymptotic case $\kappa \to \infty$. Note that one obtains different curves for different values of $\kappa\tau_2/\tau_1$.

temperature even at high temperatures, may differ very much. Therefore the use of our model gas seems to be justified if the incident shock is very strong. Finally, we give one more reason why the model gas seems to be quite suitable for the discussion of the shock reflexion problem. In papers by Buggisch (1969, 1970; see also Becker 1972) it has been shown that in one-dimensional unsteady shock reflexion the behaviour of the pressure behind the reflected shock is qualitatively the same as in real gases. (Results for CO_2 gas are given, for instance, in the paper of Johannesen *et al.* 1967.) Therefore it seems that the model gas already has properties which allow explanation of the complex behaviour of the gas flow behind the reflected shock (see also Becker & Böhme 1969, p. 108).

In order to get some insight into the behaviour of the gas flow let us first see what we should expect in the case $\tau(p_2, s_2) \equiv \tau_2 \ll \tau(p_1, s_1) \equiv \tau_1$. For this purpose we consider the process using three different length scales (see figure 2). On the first scale, both relaxation zones are infinitely thick. Then, the gas is completely frozen in both regions 1 and 2, the pressure in region 2 being p_{ff}. On the second scale, the relaxation zone behind the incident shock is still infinitely thick whereas the gas comes to equilibrium immediately behind the reflected shock. The pressure in region 2 is then p_{fe}. On the third scale, the gas is in equilibrium everywhere. In this case the pressure in region 2 is p_{ee}. Now, for the model gas, in the cases considered, $p_{fe} < p_{ff} < p_{ee}$. This being so, we must expect the pressure variation along the wall to behave qualitatively as in figure 2. Figures 3 (*a*), (*b*) and (*c*) show that, indeed, the wall pressure behaves as just predicted if τ_2 is sufficiently small compared with τ_1.

To facilitate a comparison of the different cases presented in figures 3 (*a*), (*b*) and (*c*), we have chosen as abscissa the co-ordinate

$$z = \frac{1+\kappa'/\kappa}{\Lambda_1} x$$

instead of x. This choice allows us to consider also the limiting case $\kappa \to \infty$ (figure 3) on a finite scale. The reason for this is that the width Λ_1 in x direction of the relaxation zone in region 1 is a measure of the length over which $\Delta p/p_2$ varies. All figures show the reflexion of an infinitely strong shock. It makes sense to choose this particular case because the stronger the incident shock is, the better the assumption of relatively weak influence of relaxation and of constant specific heats is realized. The fact that the strength of the shock plays an important role for the applicability of the theory can be understood easily. If for fixed ϵ the shock becomes weaker and weaker, the changes in the flow field in the relaxation zone become more and more important in comparison with the total changes (for extremely weak shocks we even get fully dispersed waves). It is therefore clear that for fixed ϵ the analysis breaks down if the shock becomes too weak. On the other hand, if we keep the shock strength constant and let ϵ tend to zero, we shall always arrive at small enough values of ϵ to make our theory applicable. Now, in a real situation, ϵ is a given quantity, and the best we can do is to choose the shock strength as large as possible. In all figures only cases with $\tau_2 \leqslant \tau_1$ are studied. The reason for this decision is that reaction rates usually increase with increasing temperature and density. The case $\tau_2/\tau_1 = 1$ has been

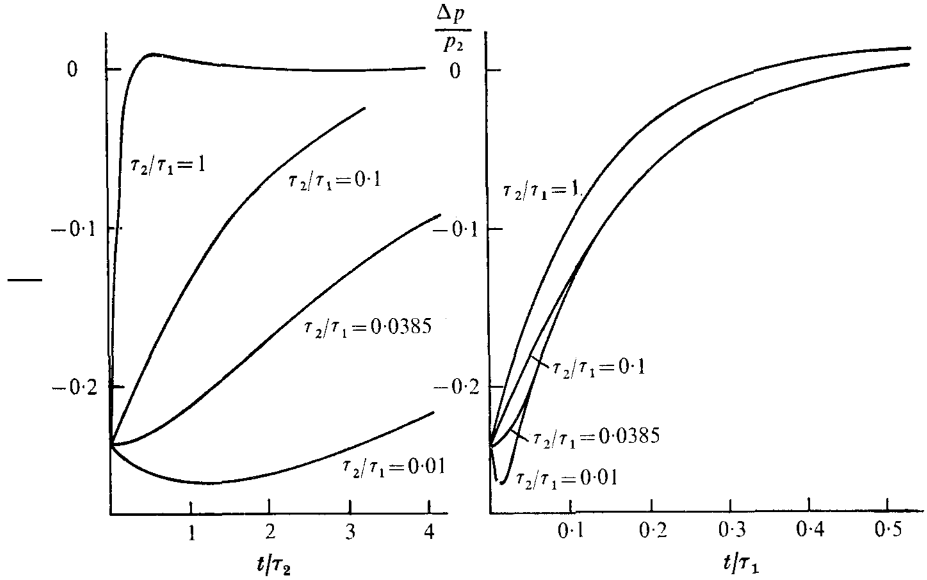

FIGURE 4. Wall pressure perturbation as a function of time for the one-dimensional unsteady reflexion of an infinitely strong shock in our model gas with $\gamma = \frac{9}{7}$.

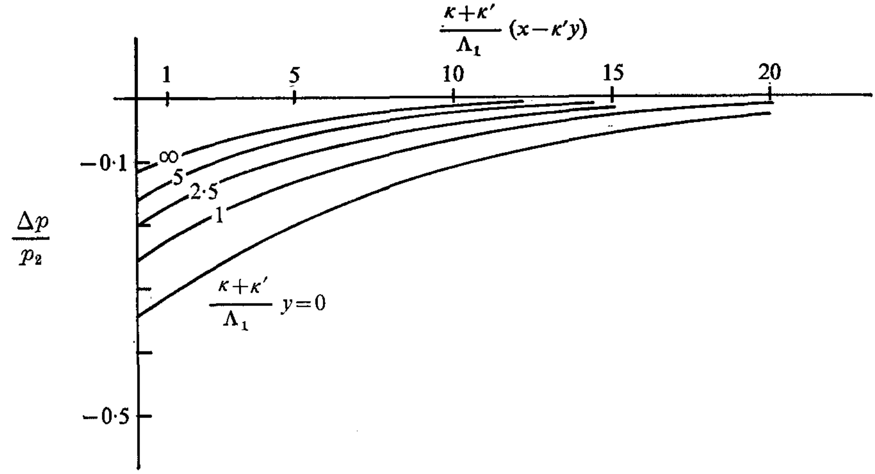

FIGURE 5. Variation of the pressure perturbation along straight lines y = constant parallel to the wall $y = 0$ for an inclination of the incident shock $\kappa = 2$ and for $\tau_2/\tau_1 = 0\cdot12$.

included although this limit can not be reached in real gases. But, since it is impossible to give an upper limit for τ_2/τ_1 which is lower than one, it seems justified to include this extreme case though it is unrealistic. In all figures with the exception of figure 4 the value $\gamma = 1\cdot5$ has been chosen for the ratio of the specific heats.

Figures 3 (*a*), (*b*) and (*c*) show the wall pressure as function of z for $\kappa = 1\cdot3$, $\kappa = 2$ and $\kappa \to \infty$ and for various values of τ_2/τ_1. The choice $\kappa = 1\cdot3$ has been made because this value of κ is close to the lowest value for which regular reflexion is possible. The case $\kappa = 2$ represents a typical case which is not extraordinary in any respect. For $\tau_2 \ll \tau_1$ the wall pressure first decreases with increasing z. This

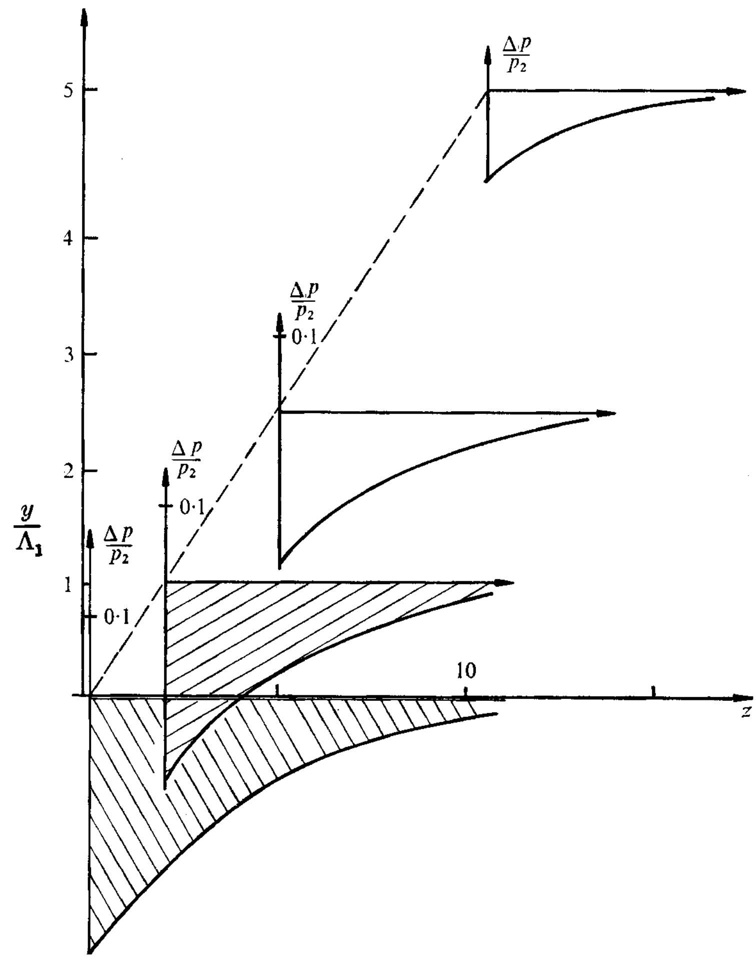

FIGURE 6. Same curves as in figure 5 with changed abscissae. The dashed line indicates the position of the reflected equilibrium shock: $x = \kappa' y$.

behaviour has been explained already. For $\tau_2 \approx \tau_1$, the wall pressure always increases for small z and even attains values larger than the asymptotic value. A similar behaviour can be observed in the one-dimensional time-dependent reflexion of a partly dispersed shock wave from a plane wall. To facilitate a comparison, figure 4, from Buggisch's papers (see also Becker 1972), has been included. It shows the wall pressure history for the one-dimensional reflexion of an infinitely strong shock in our model gas with $\gamma = \frac{9}{7}$. One more comment on figure 3 (*c*) should be made here. For $\tau_2/\tau_1 = 0$, the pressure perturbation $\Delta p/p_2$ jumps at $z = 0$ from its frozen–frozen value to its frozen–equilibrium value. This jump can be understood as limiting case of the behaviour shown by the curves of figures 3 (*a*) and (*b*) for $\tau_2 \ll \tau_1$. Figure 3 (*c*) also shows that one gets different curves for $\kappa \rightarrow \infty$ depending on how τ_2/τ_1 tends to zero at the same time. The last two figures, 5 and 6, have been included to give an impression of the entire pressure field.

REFERENCES

BAGANOFF, D. 1965 Experiments on the wall-pressure history in shock-reflexion processes. *J. Fluid Mech.* **23**, 209–228.

BECKER, E. 1972 Chemically reacting flows. *Ann. Rev. Fluid Mech.* **4**, 155–193.

BECKER, E. & BÖHME, G. 1969 *Gasdynamics, a Series of Monographs* (ed. P. P. Wegener), vol. 1. *Non-Equilibrium Flows*, part 1. Marcel Dekker.

BUGGISCH, H. 1969 Analytische Untersuchungen zur Reflexion von Verdichtungsstößen in relaxierenden Gasen. Dissertation D 17, Darmstadt.

BUGGISCH, H. 1970 Analytische Untersuchung zur Reflexion teildispergierter Wellen in relaxierenden Gasen. *Z. angew. Math. Mech.* **50**, 169–170.

CHU, B. T. 1952 On weak interaction of strong shock and Mach waves generated downstream of the shock. *J. Aero. Sci.* **19**, 433–446.

JOHANNESEN, N. H., BIRD, G. A. & ZIENKIEWICZ, H. K. 1967 Theoretical and experimental investigation of the reflexion of normal shock waves with vibrational relaxation. *J. Fluid Mech.* **30**, 51–65.

LIGHTHILL, M. J. 1949 The flow behind a stationary shock. *Phil. Mag.* **40**, 214–220.

OCKENDON, H. & SPENCE, D. A. 1969 Non-linear wave propagation in a relaxing gas. *J. Fluid Mech.* **39**, 329–345.

SMITH, J. A. 1968 Experimentally determined structure of the shock reflection process in ionizing xenon. *Phys. Fluids*, **11**, 2150–2161.

SPENCE, D. A. 1961 Unsteady shock propagation in a relaxing gas. *Proc. Roy. Soc.* A **264**, 221.